 BSc5MatC501x **Seat No :________**

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
Feb/Mar. -2021 (NEW COURSE)
Mathematical Analysis -1 & Abstract Algebra -1(Core)

Time: 1:30 Hours **Marks: 42**

Instructions:
1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q-1

(A) Attempt any **TWO**: **10**

(1) State and prove the Housedorff's principle for metric spaces.

(2) State and prove necessary and sufficient conditions for a subset of metric space to be an open set.

(3) Prove that $(\mathbb{R}, d)$ is a separable metric space , where d is the usual metric on $\mathbb{R}$.

(B) Attempt any **ONE:** **04**

(1) Let $X \neq \emptyset$ and $d: X \times X \to \mathbb{R}$ satisfies the following conditions.

(1) $d(x, y) = 0 \Leftrightarrow x = y\ ;\ \forall x, y \in X$

(2) $d(x, y) \leq d(x, z) + d(y, z);\ \forall x, y, z \in X$

Then, prove that (X, d) is a metric space.

(2) Show that $\frac{1}{4}$ is in Cantor set.

Q-2

(A) Attempt any **TWO**: **10**

(1) If f is a bounded function on $[a, b]$ and if $f \in R_{[a,b]}$, then prove that $|f| \in R_{[a,b]}$.

Also, show that $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)| dx$.

(2) State and prove the fundamental theorem of integration.

(3) For $0 \leq x \leq \frac{\pi}{2}$ show that $f(x) = cosx$ is R-integrable and find $\int_a^b cosx\ dx$.

(B) Attempt any **ONE:** **04**

(1) For $f(x) = \frac{1}{x+1}; x \in [0,1]$ and partition $P = \{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\}$, evaluate $L(P, f)$& $U(P, f)$.

(2) Evaluate $\int_0^1 e^x dx$ by using the definition of R-integration.

Q-3

(A) Attempt any **TWO**: **10**

(1) Let $f, g \in R_{[a,b]}$be such that f is continuous and g is invariant in sign on $[a, b]$. Then, prove that for some $c \in [a, b]$,

$\int_a^b f(t)g(t)dt = f(c)\int_a^b g(t)dt.$

(2) Define: Group

Show that $(G,*)$ is a group

where, $G = \mathbb{R} - \{-1\},\ \ a * b = a + b + ab, \forall a, b \in G.$

(3) Let $(G,*)$ be a group.

Then, prove that (i) Reversal Law (ii) $O(b * a * b^{-1}) = O(a), \forall a, b \in G.$

(B) Attempt any **ONE:** **04**

(1) Evaluate: $\lim_{n\to\infty} \frac{1}{n}\{e^{\frac{3}{n}} + e^{\frac{6}{n}} + \cdots + e^{\frac{3n}{n}}\}$.

(2) Prove that the set of all cube roots of unity forms a commutative group w.r.t. the binary operation of multiplication of complex numbers.

Q-4

(A) Attempt any **TWO**: **10**

(1) Prove that a non-empty subset H of a group $(G,*)$ is a subgroup of G if and only if $a * b^{-1} \in H, \forall a, b \in H$.

(2) Let G be a group and $a \in G$.
Then, prove that $H = \{a^n / \ n \in \mathbb{Z}\}$ is the smallest subgroup of G containing $'a'$.

(3) State and prove the Lagrange's theorem.

(B) Attempt any **ONE:** **04**

(1) State and prove Euler's theorem.

(2) Prove that a cyclic group with only one generator can have atmost 2 elements.

Q-5

(A) Attempt any **TWO**: **10**

(1) Prove that a subgroup H of a group G is a normal subgroup if and only if $aha^{-1} \in H, \forall a \in G, \forall h \in H$.

(2) Prove that the relation $' \cong '$ of isomorphism of groups is an equivalence relation.

(3) State and prove the Cayley's theorem.

(B) Attempt any **ONE:** **04**

(1) Show that each element of $A_n (n \geq 3)$ can be expressed as a product of cycles of length 3.

(2) Find $f^{-1}g^{-1}$ & $g^{-1}f^{-1}$ for the following permutations f & g.
$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 1 & 5 \end{pmatrix}$ & $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 5 & 4 & 2 \end{pmatrix}$

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